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Procedia Engineering 17 (2011) 179 – 196

Procedia Engineering

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The 2nd International Symposium on Aircraft Airworthiness (ISAA 2011)

Flying qualities reduction of fly-by-wire commercial aircraft with reconfiguration flight control laws

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Abstract

As to guarantee flight safety when system failures occur, modern fly-by-wire flight control systems for commercial aircraft incorporate reconfiguration control modes. While relaxed static stabilities (RSS) are utilized on most new designed fly-by-wire airliners, the flying qualities design become a major concern when reconfiguration flight control laws are developed. The aim of this paper is to analyze the reduction of flying qualities when reconfiguration laws are activated. Longitudinal and lateral-directional control laws, along with their alternative and direct modes, were developed for a commercial aircraft with relaxed longitudinal and directional static stabilities. Flying qualities assessment with reconfiguration laws were carried out based on the handling qualities rating method (HQRM), which has certification levels connected to military specifications such as MIL-STD-1797A. The analysis indicated that short period mode and Dutch roll mode are most affected in the downgrade of flight control law. It is suggested that qualities of these modes should be guaranteed as priority when reconfiguration laws are designed.

Commercial aircraft, relaxed static stability, certification, flying qualities, short period mode, Dutch roll mode

1. Introduction

Civil airworthy requirements are the minimum standards that ensure flight safety. However, they are lack of details of how a commercial aircraft should behave in flight mechanics aspect. Military flying qualities requirements, such as MIL-F-8785C, MIL-STD-1797A, not only address safety problems, but

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doi:10.1016/j.proeng.2011.10.021

also specify recommended stability and control characteristics in order to achieve higher effectiveness in completing flight tasks. As a result, military specifications are taken as a quantified reference in modern commercial aircraft certification.

As a result of the development of modern fly-by-wire systems, commercial aircraft benefit in economic performance. However, the reliability of electronic flight control systems (EFCS) is of major concern. Redundancy structures are proposed to address hardware reliability problems. Reconfiguration laws, with reduced function and flying qualities, deal with software problems [1]. Reduction of flight control law lead to downgraded handling qualities, thus compromises effectiveness in fulfilling flight tasks.

In commercial aircraft flight control systems (FCS) design, reconfiguration refers to reduction of function of FCS due to failures in redundancy systems, air data and reference units (ADIRU) or flight control computers. The technique mainly deal with lost of reference signals or calculating power in the flight computer, rather than replacing damaged control surfaces with redundant ones [2-4]. The reduced flying qualities with reconfiguration law represent minimum effectiveness and certification levels to complete control tasks.

In flight test guided AC25-7A[5], the new certification method HQRM is proposed to address flying qualities problems of EFCS equipped civil aircraft. The method set up direct relationship between airworthiness in different circumstances (i.e. different flight envelopes, atmospheric disturbance, or control system failures). Failure in FCS is the only concern in this paper and is the only circumstance to be considered. According to detailed requirement, FCS augmented aircraft should fulfill satisfactory level (which is equivalent to level 1 in MIL standard) when no failures occur. The minimal handling qualities requirement demand that FCS should achieve adequate level, which is equivalent to level 2 in military standard, when failures and control law reconfiguration occur. Thus, level 2 in MIL-STD-1797A[6] is the rudimentary requirement that should be used to certificate FCS.

In this paper, a commercial airliner with RSS design is modeled. Longitudinal and lateral-directional flight control laws, with their alternative and direct modes, are designed. Certification of FCS are mainly carried out using HQRM method, thus MIL-STD-1797A is used to determine qualities of the augmented aircraft. The assessment is of short period mode responses and lateral-directional mode responses. Comparisons are made among qualities of different levels of control laws.

Nomenclature

β , β_{cmd} Sideslip angle and sideslip command

δ_{a} Deflection angle of aileron

δ_{e} Deflection angle of elevator

δ_{r} Deflection angle of rudder

δ_{H} Deflection angle of horizontal stabilizer

ζ_{DR} Damping ratio of bare airframe Dutch roll mode, or closed loop Dutch roll damping ratio of an augmented aircraft

$\zeta_{\mathrm{DR}} \cdot \omega_{\mathrm{nDR}}$ Bare airframe of closed loop Dutch roll damping

ζ_{sp}' Desired short period mode damping ratio

λ_{DR} Desired closed loop pole of Dutch roll mode

λ_R	Desired closed loop pole of roll mode
$\lambda_{Roll,err}$	Desired pole of nuisance mode introduced by integrator in roll control path
$\lambda_{Yaw,err}$	Desired pole of nuisance mode introduced by integrator in yaw control path
λ_{DR}	Desired closed loop pole of Dutch roll mode
$\mathbf{v}_{DR}$	Eigenvector of closed loop Dutch roll mode
$\mathbf{v}_R$	Eigenvector of closed loop roll mode
$\mathbf{v}_{Roll,err}$	Eigenvector of closed loop nuisance mode introduced by roll axis integrator
$\mathbf{v}_S$	Eigenvector of closed loop spiral mode
$\mathbf{v}_{Yaw,err}$	Eigenvector of closed loop nuisance mode introduced by yaw axis integrator
τ_β , τ_ϕ	Sideslip and bank angle time delay of the equivalent system
ϕ , $\dot{\phi}$	Bank angle and bank angle rate
ϕ , $\dot{\phi}$	Bank angle and bank angle rate
$\dot{\phi}_{cmd}$	Bank angle rate command
ψ_β	Phase lag of sideslip motion in Dutch roll oscillation
ω_1	Adjustable pole in closed loop longitudinal dynamics model
ω_2	Adjustable zero in closed loop longitudinal dynamics model
ω_{nDR}	Dutch roll mode natural frequency of bare airframe, or closed loop aircraft
ω_{nsp}	Natural frequency of bare airframe short period mode
ω_{nsp}'	Desired short period mode natural frequency of augmented aircraft
ω_{sp}	Equivalent short period natural frequency
$\omega_{sp}T_{\theta 2}$	Product of equivalent short period frequency and time delay between pitch attitude response and flight path response
h	Altitude
g_0 , g_1 , g_2	Coefficients in the numerator and denominator of longitudinal closed loop aircraft transfer function
h	Altitude
n/α	Normal acceleration sensitivity

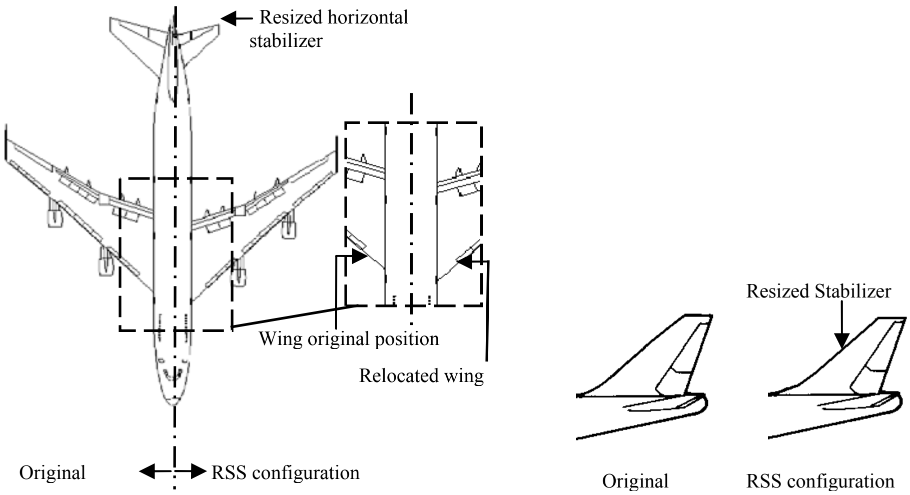
n_y	Lateral load factor
n_z	Normal load factor
$n_{z,cmd}$	Normal load factor command
n_z'	Normal load factor at instantaneous center of rotation
p	Roll rate
p_{OSC}, p_{AV}	Roll rate oscillation amplitude and average roll rate triggered by step lateral stick force input
q	Pitch rate
$\lvert \dot{q} \cdot F_s^{-1} \rvert_{max}$	Maximum frequency response amplitude ratio of the pitch acceleration to pitch control force
r	Yaw rate
$C*$	C-star, a blend of normal load factor and pitch rate
F_s	Longitudinal control force
F_a, F_r	Lateral and directional control force
$F_s \cdot n_z^{-1}$	Longitudinal control force gradient
$K_{\beta,ny}$	Coefficient that transfer lateral load factor to sideslip angle
$K_{\delta eNz}$	Normal load factor feedback gain in longitudinal normal law
K_ϕ	Coefficient in lateral-directional low order equivalent systems transfer functions
K_{fa}	Roll axis portion gain in lateral-directional law
K_{fr}	Yaw axis portion gain in lateral-directional law
K_{Ia}	Roll axis integrator gain in lateral-directional law
K_{Ir}	Yaw axis integrator gain in lateral-directional law
K_{ny}	Lateral load factor feedback gain in lateral-directional law
K_r	Yawrate feedback gain in lateral-directional law
K_{ARI}	Aileron-rudder-interconnection gain
Ma	Mach number
$T_{\beta 1}$, $T_{\beta 2}$, $T_{\beta 3}$	Time constant of lateral-directional low order equivalent system transfer function
$T_{\theta 2}$	Time between pitch attitude response and flight path response
T_w	Time constant of yaw damper washout filter

2. Aircraft Modeling

2.1. Bare airframe model

Longitudinal static stability can be reduced by following means: (a) Relocating wing at a forward position; (b) Reducing size of horizontal stabilizer. The former brings both aerodynamic center of the aircraft and center of gravity (CG) forward, while excursion of aerodynamic center is larger. The latter further reduces longitudinal static stability. Directional static stability can be relaxed by reducing size of vertical stabilizer. Resizing of the fins can reduce cruise drag, thus improve economic performance.

Based on a Boeing 747 airliner [7], a commercial jet with RSS design was modeled. Modifications to wing position (1.03m ahead of original position), horizontal stabilizer size (38.5m^2 smaller than original) and vertical stabilizer size (5.1m^2 smaller than original) result in relaxed longitudinal stability (5%, at cruising altitude, Ma =0.8, $h = 12192\text{m}$) and directional stability ($C_{n\beta}$ is 15% smaller than original). The modifications are shown in figure 1.



(a) Plan view; (b) Side view of rear fuselage sections

Fig. 1. RSS modification of the original B747 model

Due to the relaxation of longitudinal and directional static stability, the pitch and yaw stiffness of bare airframe are reduced. The degraded flying qualities associated with these modifications are shown in table 1. The aircraft is cruising at 6096m at Mach number 0.5.

Table 1. Mode characteristics decays of RSS aircraft

	B747 original	B747 RSS
	Short period mode (longitudinal)	
ω_{nsp}	1.06	0.60
n/α	6.02	6.02
CAP	0.115	0.060
Military standard level	1	2

	Dutch roll mode (directional)	
ζ_{DR}	0.086	0.045
ω_{nDR}	0.900	0.666
$\zeta_{DR} \cdot \omega_{nDR}$	0.077	0.030
Military standard level	2	3

Since the aerodynamic center gets closer to CG in the RSS airplane, decays appears in short term mode's natural frequency. While ω_{nsp} drops from 1.06 to 0.60, CAP drops to 0.060 with handling qualities ratings degraded.

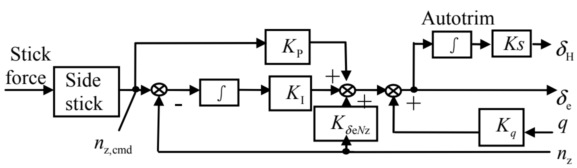
The reduction of vertical fin's area leads to smaller tail volume, thus Dutch roll frequency decreases along with damp ratio of the oscillation mode. The original aircraft already suffers from insufficient Dutch roll damping, Reductions of ζ_{DR}, ω_{nDR} and $\zeta_{DR} \cdot \omega_{nDR}$ lead to further deficiencies. Therefore, Dutch roll qualities of the modified aircraft only satisfy level 3 requirements, the airplane is barely controllable. Therefore, flying qualities design is the key to airworthiness certification for RSS commercial aircraft.

2.2. Longitudinal control law

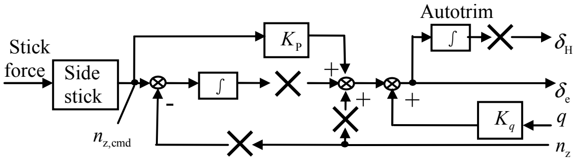
Longitudinal and lateral-directional flight control laws, depicted in figure 2 and 3, are developed to improve the degraded flying qualities. It should be noted that the control laws used in Airbus A320/330 aircraft has been taken as the major design references [4].

Longitudinal law consists of 3 modes: (a) Normal mode provides augmentations while enabling pilots to issue normal load factor or *C** commands, where *C** is a blended command of load factor and pitch rate, which is used in low speed region. Also incorporated in the mode are flight envelope protection function and a gust alleviation law that improves flight safety and riding qualities; (b) Alternative mode, also known as ALT, provides augmentations and load factor/*C** command functions while flight envelope protection and gust alleviation functions are lost. The ALT mode is entered when redundant systems failures occur; (c) Direct mode, known as DIR, is entered when severe failures occur (e.g., engine flameout, failures in ADIRU or total failures in flight control computers). In DIR mode, direct surface control is activated with rudimentary damping feedback[8, 9].

Since flight envelope protection and gust alleviation functions has little impact on handling qualities in small disturbance condition, these functions are ignored in this paper. Therefore, longitudinal control laws in normal mode and ALT mode share the same structure (as depicted in figure 2a). It is noteworthy that the airplane is cruising at Mach 0.5 at high altitude, with its control law in load factor mode. DIR law is shown in figure 2b.



(a) Normal and alternative (ALT) law



(b) Direct (DIR) law

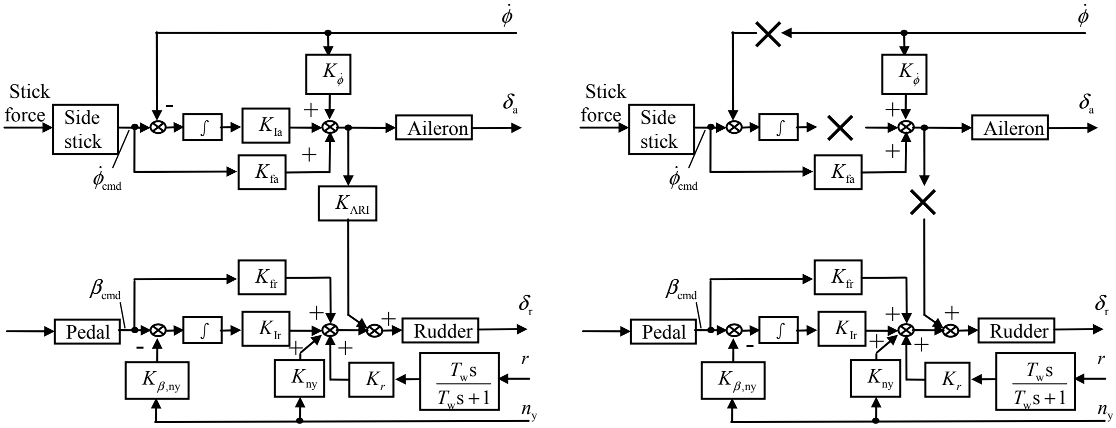
Fig 2. Layout of longitudinal control law

The normal/ALT law allows pilots to control normal load factor of the airplane. Stick force is interpreted into load factor command. The inner loops consist of a pitch damper and a stability augmentation feedback, whose gains are K_q and $K_{\delta eNz}$ respectively. The outer loop is made up of a feedback loop for load factor command, and portion-integration (PI) path, whose gains are K_P and K_I respectively. The portion path supplies control gain, while the integration path eliminates static error of load factor. Parallel to elevator path is a horizontal stabilizer control path (also known as 'autotrim' path), which has an integrator, whose gain is K_s, to unload the hinge moment of the elevators.

The DIR law is literally a normal law with all feedbacks (except pitch damper), stick shaping and auto-trim path removed. Stick displacement generate an elevator deflection command directly, which is augmented by pitch damper command.

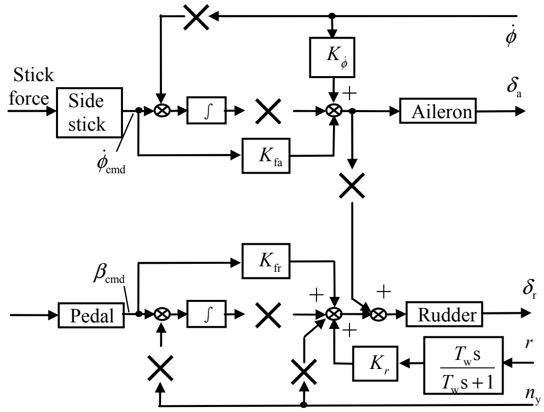
2.3. Lateral-directional law

The lateral-direction law is depicted in figure 3.



(a) Normal law

(b) Alternative law



(c) Direct law

Fig 3. Layout of lateral-directional control law

As shown in figure3a, the lateral-directional law is developed to facilitate pilots' control on bank angle rate and sideslip angle. The control law also improves lateral-directional mode qualities with its stability augmentation feedbacks.

The lateral law consists of a roll-axis command path and a yaw-axis command path. The roll axis path is augmented with a roll damper (whose gain is $K_{\dot{\phi}}$), while the yaw-axis command path is augmented with a yaw damper (whose gain is K_r) and a lateral load factor feedback (whose gain is K_{ny}). Command augmentation in each control path is achieved by using a portion-integration (PI) structure. In roll axis path, bank angle rate command $\dot{\phi}_{\mathrm{cmd}}$ is amplified by gain K_{fa} while static error of bank angle rate is eliminated by the integrator, K_{Ia} . In yaw axis command path, sideslip command β_{cmd} is amplified by K_{fr} while static error is eliminated with integrator (K_{Ir}). Aileron-rudder interaction (ARI) path, whose gain is K_{ARI} , is added between the 2 control paths as to reduce sideslip motion.

ALT mode (figure 3b) of lateral law is entered when redundancy failures in FCS occur. In this mode, the PI structure that controls bank angle rate is lost, along with ARI path. Lateral stick controls aileron with a direct link, whose gain is K_{fa} , while roll damper is still available. Yaw axis path remains the same as in normal mode, i.e., pedals still controls sideslip angle.

In DIR mode (figure 3c), PI structures of both command path are lost. Sidestick and pedal displacements are amplified directly to generate aileron and rudder commands, which are augmented by rudimentary damper signals.

3. Analysis of Closed Loop Longitudinal Handling Qualities

3.1. Normal law and alternative law

One can use a transfer function to represent short period dynamics of an conventional aircraft with C* command law[1]. Similar work can be done to a load factor demand law.

With phugoid mode and actuator dynamics ignored, the augmented aircraft can be represented by the following transfer function.

$$\frac{\Delta n_z(s)}{\Delta n_{z,\mathrm{cmd}}(s)}=\frac{g_0}{\omega_2}\cdot\frac{s+\omega_2}{s^3+g_2 s^2+g_1 s+g_0} \tag{1}$$

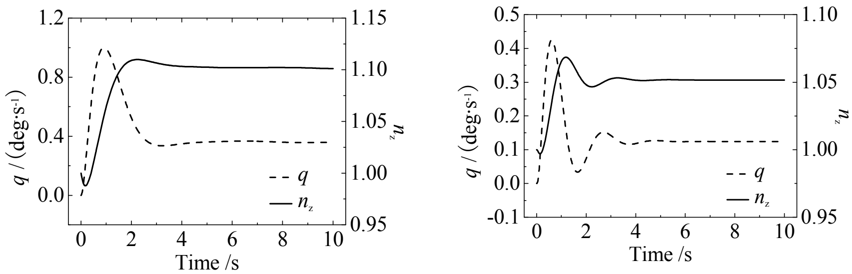
As shown in equation 1, the closed loop aircraft has 1 zero and 3 poles. Due to the presence of integrator in command path, the static state error of normal load factor is zero. The intended load factor transfer function can be written in a short period oscillation form, as shown in equation 2.

$$\frac{\Delta n_{\mathrm{z}}(\mathrm{s})}{\Delta n_{\mathrm{z,cmd}}(\mathrm{s})}=\frac{\omega_1\omega'_{\mathrm{nsp}}}{\omega_2}\frac{(\mathrm{s}+\omega_2)}{(\mathrm{s}+\omega_1)(\mathrm{s}^2+2\zeta'_{\mathrm{sp}}\omega'_{\mathrm{nsp}}\mathrm{s}+\omega'^{\,2}_{\mathrm{nsp}})} \tag{2}$$

Two conjugate closed loop poles and a spare pair of zero and pole are present in this form. ζ'_{sp} is the intended short period damping ratio, ω'_{nsp} is the intended short period natural frequency in rad/s, ω_1 and ω_2 (in rad/s) are adjustable pole and zero.

It takes an enormous effort to improve the short period natural frequency of an RSS transport aircraft because of its poor pitch stiffness. Large poles and zeros may lead to high gains in FCS and insufficient elevator authorities. Therefore a moderate value of ω'_{nsp} (1.5rad/s), rather than a large one, is chosen. Damping ratio is set as 0.7. ω_1 and ω_2 are set with the same value (0.5rad/s) as to cancel each other. Once the intended values in the transfer function are set, control law gains can be calculated by comparing the actual and intended form of short-term response.

As mentioned above, the aircraft is cruising with its longitudinal control law commanding normal load factor. As shown in figure 4, pitch axis step response of augmented aircraft resembles that of a classical aircraft.



(a) Augmented aircraft (b) Classical aircraft

Fig 4. Pitch axis step response of augmented and classical aircraft

It should be noted that figure 4a depicts the response of the augmented aircraft modeled in this paper, while figure 4b shows the response of a classical aircraft. Despite the differences in frequency and damping, the load factor and pitch rate response of augmented aircraft shares similar oscillation frequencies. This characteristic is also found on classical aircraft. Therefore, a good low order equivalent system (LOES) fitting of the augmented aircraft can be achieved, as shown by equation 3.

$$\begin{cases}\dfrac{q(\mathrm{s})}{F_{\mathrm{s}}(\mathrm{s})}=\dfrac{0.0029(\mathrm{s}+0.4045)}{\mathrm{s}^2+2.0533\mathrm{s}+2.8298}\mathrm{e}^{-0.1036\mathrm{s}}\\[2ex]\dfrac{n'_{\mathrm{z}}(\mathrm{s})}{F_{\mathrm{s}}(\mathrm{s})}=\dfrac{0.0179}{\mathrm{s}^2+2.0533\mathrm{s}+2.8298}\mathrm{e}^{-0.0510\mathrm{s}}\end{cases} \tag{3}$$

LOES parameters can be used to determine the qualities of short term mode. From equation 2, we know that the classic form of augmented aircraft has a short period natural frequency of 1.68rad/s, damping ratio of 0.61. The delay of path response after pitch attitude response is 0.4045s. The pure delay of pitch response to stick input is 0.1036s. Short period pitch response requirements are given regarding the natural frequency (ω_{sp}), damping ratio (ζ_{sp}) and delay ($T_{\theta 2}$), as shown in figure 5.

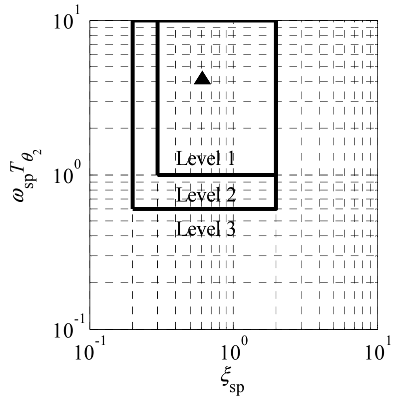


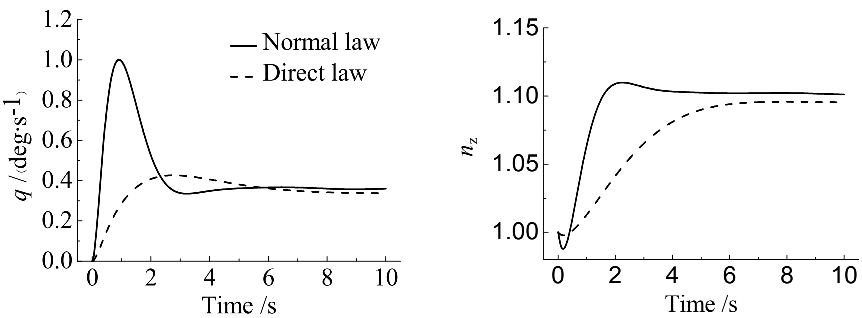
Fig 5. Short period pitch response level of augmented aircraft

It can be seen from figure 5 that the equivalent damping, natural frequency and time delay between path and pitch rate response satisfies level 1 requirement. Pure delay of transport category aircraft should be less than 0.2s if it is to reach level 1[10]. Thus, a delay of 0.1036s is satisfactory.

3.2. Direct law

The direct mode of longitudinal law has only one loop, the pitch damper. Therefore its gain K_q can be determined with by calculating the closed loop transfer function. The command path gain K_p is chosen to bring about least change in stick force gradient.

Decays in response with direct law are shown in figure 6.



(a) Pitch rate (b) Normal load factor

Fig 6. Pitch step response with normal law and direct law

When DIR law is activated, it takes longer time for pitch-rate and load factor to reach their static state value. The size of overshoots also drops significantly. The lost of load factor signals leads to lower pitch stiffness of the closed loop aircraft, therefore the rise time increases. Due to the absence of pitch static stability augmentation, the closed loop aircraft is over damped. Hence the smaller overshoots.

Closed loop aircraft with DIR law can also be approximated with LOES models, as shown in equation 4.

$$\begin{cases} \dfrac{q(\mathrm{s})}{F_{\mathrm{s}}(\mathrm{s})} = \dfrac{0.0009(\mathrm{s}+0.3903)}{\mathrm{s}^2+1.008\mathrm{s}+0.4808}\mathrm{e}^{-0.0964\mathrm{s}} \\ \dfrac{n_z'(\mathrm{s})}{F_{\mathrm{s}}(\mathrm{s})} = \dfrac{0.0179}{\mathrm{s}^2+1.008\mathrm{s}+0.4808}\mathrm{e}^{-0.0431\mathrm{s}} \end{cases} \tag{4}$$

Decays of longitudinal flying qualities can be examined with multiple criteria. Military standard MIL-STD-1797A provided three categories of short period qualities criteria: (a) criteria using LOES dynamics, e.g. control anticipation parameters (CAP), combination of ω_{sp} and $T_{\theta 2}$; (b) frequency domain related criteria that specifies actual high order system (HOS) dynamics, e.g. bandwidth and Neal-Smith criteria; (c) time domain related criteria using actual HOS dynamics, e.g. transient peak ratio criteria and Gibson's drop back criterion. It is suggested one use as many of these criteria as possible to reveal underlying defects of FCS. However, it is more practical to use a limited selection of these criteria since most of them are related with CAP, damping ratio and time delays. Therefore, LOES parameters criteria ($\omega_{sp}T_{\theta 2}$) , which is an alternative form of the CAP criteria, and transient peak ratio criterion are chosen. Decays of flying qualities can be examined with the two criteria, which are relatively easy to use.

LOES parameters with normal law and DIR law are depicted in figure 7.

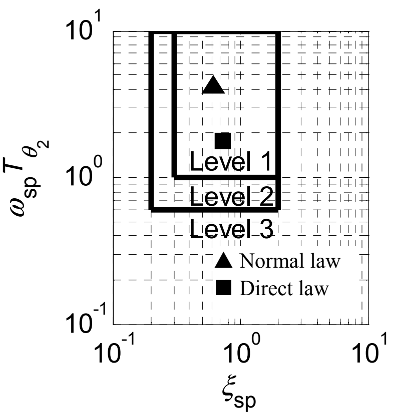


Fig 7. Decays of flying qualities in longitudinal direct law

$\omega_{sp}T_{\theta 2}$ falls from 4.158 to 1.176 when control augmentations are lost. Although the parameter still satisfies level 1 requirement, it is very close to the boundary that divides level 1 and level 2.

The transient peak ratio criterion further reveals decays in CAP, which is shown in table 2.

Table 2. Pitch rate step response levels with DIR law

Response parameters	Level 1 requirement	Actual value
Equivalent time delay /s	≤ 0.12	0.11
Rise time Δt /s	$0.017 \leq \Delta t \leq 0.776$	1.20
Transient peak ratio	≤ 0.30	0
$(F_s \cdot n_z^{-1}) \cdot \lvert \dot{q} \cdot F_s^{-1} \rvert_{max}$	≤ 3.6	0.035

In this criterion, equivalent delay is the delay of HOS response to pilot commands in time domain. It is close in meaning to the delay derived from LOES method. Rise time of pitch rate has a strong connection with CAP of the augmented aircraft[6]. The shortest acceptable rise time corresponds to highest acceptable CAP value and vice versa. The transient peak ratio corresponds to short period damp ratio ζ_{sp}. $(F_s \cdot n_z^{-1}) \cdot \lvert \dot{q} \cdot F_s^{-1} \rvert_{max}$ is the product of control force gradient $F_s \cdot n_z^{-1}$, and the maximum frequency response amplitude ratio of the pitch acceleration to pitch control force $\lvert \dot{q} \cdot F_s^{-1} \rvert_{max}$. Literally, it is equal to CAP.

As can be seen in table 2, the rise time (1.20s) is lager than 0.776, the upper boundary of rise time for level 1. That is to say, the CAP of aircraft with DIR law is too small to satisfy level 1 requirement. This is further confirmed by parameter $(F_s \cdot n_z^{-1}) \cdot \lvert \dot{q} \cdot F_s^{-1} \rvert_{max}$, whose value is 0.035, falls below CAP level 1 limit.

Although the LOES parameters criterion suggests the airplane with DIR law satisfies level 1 requirement, it did reveal the decays in short period frequency and CAP. The time domain pitch rate response criterion also reveals that downgrading of FCS brings about reduction of CAP qualities. It can be concluded that FCS failures has triggered reduction of longitudinal short term qualities. It is noteworthy that level 2 requirements, which correspond to “adequate category” requirements in civil aircraft certifications, are to be satisfied when system failures occur. The DIR law proposed above is able to fulfill a level 2 requirement and therefore satisfies HQRM airworthy certificate requirement.

As stated above, relaxed longitudinal static stability leads to significant reduction of pitch stiffness of the bare airframe. This brings about challenges in the flying qualities design of longitudinal direct control law. It is recommended that RSS design effort should not exceed the ability of longitudinal DIR law to fulfill level 2 requirement in MIL-STD-1797A, which is correspond to “adequate” level in HQRM method.

4. Analysis of Closed Loop Lateral-Directional Handling Qualities

4.1. Normal law

The lateral-directional control law is designed to improve the closed loop mode qualities and facilitate pilots’ control on bank angle rate and sideslip.

The gains in the lateral law are selected using eigen- structure assignment (EA) method. It should be noted that bank angle rate feedback is only available in roll control path, while feedbacks of yawrate and lateral load factor exist in yaw axis path only. Therefore the EA method used here is based on partial state feedback [11]. A linear model of the HOS is built. Interconnections between roll axis and yaw axis, first order dynamics of aileron and rudder actuators, integrators dynamics in both command paths are included in the model. Eigenvalues and eigenvectors of the closed loop system are selected to guarantee satisfactory lateral mode dynamics. It is noteworthy that “nuisance” modes such as spiral can be eliminated by bank angle rate PI command structure. When lateral control force is zero, the bank angle will remain unchanged, hence the absence of closed loop spiral mode. One should also note that closed loop "nuisance" modes associated with integrators in roll axis, $\lambda_{\text{Roll, err}}$, and yaw axis, $\lambda_{\text{Yaw, err}}$, are also introduced. Therefore, the augmented aircraft features 2 classical modes: roll mode and Dutch roll mode, as well as some nuisance modes. The desired closed loop eigenvalues are shown in equation 5.

$$\begin{cases} \lambda_{\text{R}} = -2.5 \text{ ..... roll mode} \\ \lambda_{\text{S}} = -0.3 \text{ ...... spiral 'nuisance' mode} \\ \lambda_{\text{DR},1} = -0.5 + \text{i} \text{ ... Dutch roll mode} \\ \lambda_{\text{DR},2} = -0.5 - \text{i} \text{ ... Dutch roll mode} \\ \lambda_{\text{Roll, err}} = -0.5 \text{ .... bank angle rate error} \\ \lambda_{\text{Yaw, err}} = -0.5 \text{ .... sideslip error} \end{cases} \tag{5}$$

In equation 4, eigenvalues of roll mode and Dutch roll mode are selected to meet the level 1 requirement in MIL-STD-1797A. The pole, λ_{S}, now represents a "nuisance" mode, is set at . As a result, the mode can converge rapidly without interfering with strong modes. The other "nuisance" modes, whose closed loop poles are and , are assigned to for the same reason.

Eigenvectors represents the amplitude and phase make up of each closed loop mode. The desired eigenvectors of the strong modes are listed in equation 6, while the nuisance mode vectors are listed in equation 7.

$$\underbrace{\mathbf{v}_{\text{R}} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \times \\ \times \\ \times \end{bmatrix}}_{\text{roll mode}} \quad \underbrace{\mathbf{v}_{\text{DR}} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \\ \times \\ \times \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ \times \\ \times \end{bmatrix}}_{\text{Dutch roll}} \quad \begin{matrix} ...\beta \\ ...p \\ ...r \\ ...\phi \\ ...\int(\dot{\phi}_{\text{cmd}} - \dot{\phi}) \\ ...\int(\beta_{\text{cmd}} - \beta) \end{matrix} \tag{6}$$

In each vector shown in equation 6, elements from top to below represents sideslip, roll rate, yaw rate, bank angle, integrated bank angle rate error and integrated sideslip error respectively. The desired roll mode is a single degree of freedom (DOF) roll motion. Therefore, state variables that represent sideslip and yaw motion are set as zero (this suggests the two motions should be suppressed). Rollrate, the state variable which is closest to command roll axis variable , is set 1. Thus, roll rate becomes the major state variable in this mode and has an larger amplitude than sideslip and yawrate. The crosses in the vector indicates roll angle, integrated command errors in both control paths are left unprocessed, since they do not interfere with the make up of intended closed loop roll mode.

Desired closed loop Dutch roll mode features oscillation that consists of a sideslip and yaw motion, as well as the roll motion which is triggered by yaw motion (this is suggested by the phase make up) [12]. Therefore there are a pair of conjugate vectors which features phase relationships among state variables (only one vector is shown in equation 3). One should note Dutch roll mode of a classical aircraft has a roll motion component larger than yaw and sideslip. Since the state feedbacks are not present in both command paths, the control law does not support full state feedback. It is therefore more practical to decrease the roll motion amplitude, rather than trying to remove it entirely. As a result, the amplitude of roll motion is identical to sideslip and yaw motion in the proposed eigenvector.

$$\underbrace{\mathbf{v}_{\mathrm{S}}=\begin{bmatrix}0\\ \times\\ 0\\ 1\\ \times\\ \times\end{bmatrix}}_{\text{nuisance spiral}}\quad \underbrace{\mathbf{v}_{\mathrm{Roll,err}}=\begin{bmatrix}0\\ \times\\ \times\\ \times\\ 1\\ 0\end{bmatrix}}_{\text{integration of bank angle rate error}}\quad \underbrace{\mathbf{v}_{\mathrm{Yaw,err}}=\begin{bmatrix}\times\\ 0\\ \times\\ \times\\ 0\\ 1\end{bmatrix}}_{\text{integration of sideslip error}}\begin{array}{l}\ldots\beta\\ \ldots p\\ \ldots r\\ \ldots\phi\\ \ldots\int(\dot{\phi}_{\mathrm{cmd}}-\phi)\\ \ldots\int(\beta_{\mathrm{cmd}}-\beta)\end{array}\tag{7}$$

Although spiral mode becomes a "nuisance" mode, it is desirable to remove sideslip and yaw components as to further decouple roll and yaw motions. As can be seen in equation 6, the other two "nuisance" modes are introduced by adding integrators to lateral and directional command path. The "1"s and "0"s in the two vectors indicate designers' try to decouple the lateral-directional motions.

Now the settings of gains are complete. The lateral-directional flying qualities of the augmented aircraft can be examined through: (a) mode characteristics; (b) rollrate and sideslip response to step roll input.

Lateral-directional FCS has changed closed loop mode characteristics of the airplane significantly, while introducing additional higher-order modes. Therefore, LOES is introduced to approximate the actual bank and sideslip response to sidestick and pedal inputs of the high order aircraft. The lateral LOES takes the form of what is shown in equation 8[6].

$$\begin{cases}\dfrac{\phi(\mathrm{s})}{F_{\mathrm{a}}(\mathrm{s})}=\dfrac{K_{\phi}\left(\mathrm{s}^2+2\zeta_{\phi}\omega_{\phi}\mathrm{s}+\omega_{\phi}^{2}\right)}{\left(\mathrm{s}+1/T_{\mathrm{R}}\right)\left(\mathrm{s}+1/T_{\mathrm{S}}\right)\left(\mathrm{s}^2+2\zeta_{\mathrm{DR}}\omega_{\mathrm{nDR}}\mathrm{s}+\omega_{\mathrm{nDR}}^{2}\right)}\mathrm{e}^{-\tau_{\phi}\mathrm{s}}\\[2ex] \dfrac{\beta(\mathrm{s})}{F_{\mathrm{r}}(\mathrm{s})}=\dfrac{K_{\beta}\left(\mathrm{s}+1/T_{\beta1}\right)\left(\mathrm{s}+1/T_{\beta2}\right)\left(\mathrm{s}+1/T_{\beta3}\right)}{\left(\mathrm{s}+1/T_{\mathrm{R}}\right)\left(\mathrm{s}+1/T_{\mathrm{S}}\right)\left(\mathrm{s}^2+2\zeta_{\mathrm{DR}}\omega_{\mathrm{nDR}}\mathrm{s}+\omega_{\mathrm{nDR}}^{2}\right)}\mathrm{e}^{-\tau_{\beta}\mathrm{s}}\end{cases}\tag{8}$$

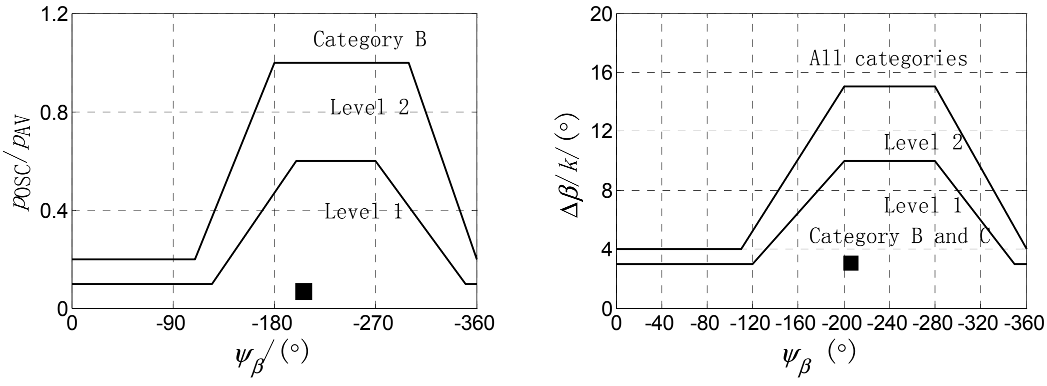
Level 1 recommendations are given in MIL-STD-1797A regarding roll mode time constant, T_{R}, spiral mode stability and yaw axis dynamic response. The closed loop airplane satisfies level 1 requirement as a result of control augmentation, as can be seen in table 3.

Table 3. Lateral-directional dynamics response levels (normal law, cruising)

Response parameters	Level 1 requirement	Actual value
Roll time constant	≤ 1.4	0.49 (level 1)
Spiral mode double time /s	≥ 20	Neutral (level 1)
Dutch roll natural frequencies /(rad/s)	≥ 0.4	1.56 (level 1)
Dutch roll damping ratio	≥ 0.08	0.34 (level 1)
Dutch roll damping $\zeta_{\mathrm{DR}}\cdot\omega_{\mathrm{nDR}}$ /(rad/s)	≥ 0.15	0.53 (level 1)

Roll time constant of the closed loop airplane is 0.49s, which corresponds to a closed loop eigenvalue of -2.04. Closed loop Dutch roll damping and natural frequency are 0.34 and 1.5rad/s relatively. Characteristics of both strong modes are close to design intentions. It is noteworthy that LOES approach supposes high order airplane responses to take a classical form. Therefore a neutral spiral mode is introduced in the classical model. In fact, the mode has already been eliminated by the roll-axis path PI command structure.

Time domain criteria are introduced to further examine roll and sideslip response to lateral stick input. As can be seen in figure 8, the roll rate oscillation and associated sideslip are within level 1 boundaries [6].



(a) Rollrate oscillation levels (b) Sideslip amplitude levels

Fig 8. Rollrate and sideslip oscillation levels (lateral-directional normal law)

Therefore, the lateral-directional control law has achieved satisfactory lateral directional handling qualities. Thus, airworthiness requirements for normal laws are automatically satisfied.

4.2. Alternative and direct law

As is described in section 2.3, the lateral alternative law, or ALT law (depicted in figure 3b), consists of a roll axis direct law command path and a yaw axis path with full command augmentation. The removal of bank angle rate feedback path and the integrator in roll axis command path is equivalent to removal of a bank angle feedback. Therefore roll stiffness of the closed loop airplane is reduced. All the gains in ALT law remain unchanged as in normal law, except for the gain in direct link of roll axis, .

The lateral direct law, or DIR law (depicted in figure 3c), is made up of 2 direct control paths as well as rudimentary dampers. The sideways accelerometer signal is lost. Therefore yaw stiffness compensator, whose gain is K_{ny}, is inactive. Therefore yaw damper gain, K_r, is reduced as to avoid a significant reduction of yaw stiffness.

The close loop mode qualities of lateral system in ALT and DIR mode can be assessed using equivalent system technique as well as time domain criteria.

LOES parameters are calculated to approximate lateral system in ALT and DIR modes. The closed loop dynamic characteristic parameters are shown in table 4.

Table 4. Lateral-directional dynamics response levels (reconfiguration law, cruising)

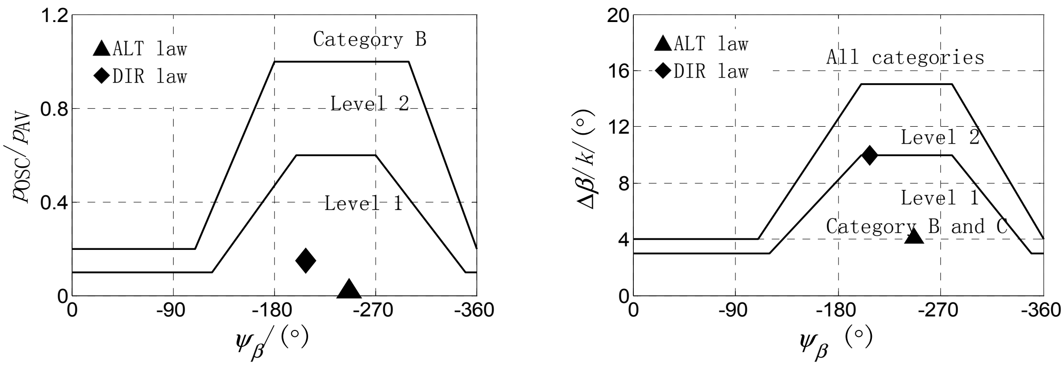
Response parameters	ALT law	DIR law
Roll time constant	0.61 (level 1)	0.58 (level 1)
Spiral mode double time /s	58.2 (level 1)	Converge (level 1)
Dutch roll natural frequencies /(rad/s)	1.628 (level 1)	0.712 (level 1)
Dutch roll damping ratio	0.444 (level 1)	0.318 (level 1)
Dutch roll damping $\zeta_{DR} \cdot \omega_{nDR}$ /(rad/s)	0.723 (level 1)	0.446 (level 1)

Although the reduction of lateral-directional control functions does not lead to a significant decay in closed loop mode characteristics (i.e. level 2), it does has impact on spiral mode and Dutch roll stabilities. With the removal of PI structure in roll command path, the spiral mode re-appears. The closed loop Dutch roll frequency and damping in ALT law are close to that of normal law, while both the damping and frequency are significantly reduced in DIR law.

The spiral stability depends on the opposite acts of roll static stability and yaw static stability. Roll stiffness provides a restore roll moment when a bank angle is established, while the yaw stiffness causes the airplane to roll further. Therefore, the spiral mode is divergent when roll stiffness is insufficient. As is mentioned earlier, the removal of lateral PI control structure leads to weakened roll stiffness, while the augmented yaw stiffness is still strong. This leads to a divergent spiral mode. In DIR mode, both the lateral PI structure and yaw stiffness augmentation are lost. Therefore, both lateral and directional static stabilities are weakened. This re-balances the two opposing acts, thus lead to a convergent spiral mode.

In normal law, the frequency and damping demands of closed loop Dutch roll are satisfied mainly with lateral load factor feedback and yaw damper. Load factor feedback compensates yaw stiffness, but it reduces Dutch roll damp ratio. The yaw damper acts in the opposite way. While yaw stiffness compensator is lost in DIR law, the gain of yaw damper should be reduced to avoid a significant reduction of yaw stiffness. It is therefore that both yaw damping and yaw stiffness have dropped significantly in DIR law.

Besides closed loop mode characteristics, the time responses of rollrate and sideslip to a lateral stick step input are also examined. See figure 9.



(a) Rollrate oscillation levels (b) Sideslip amplitude levels

Fig 9. Rollrate and sideslip oscillation levels of lateral step input (alternative and direct laws)

Rollrate oscillation levels (figure 9a) indicate no significant reduction in Dutch roll damping with ALT law, while the amplitude of the oscillation increases in DIR law as a result of weakened Dutch roll damping. However, both reconfiguration laws satisfy level 1 requirement.

Sideslip amplitude levels (figure 9b) indicate no significant reduction of yaw stiffness in ALT law, as the maximum sideslip $\Delta\beta / k$ satisfies level 1 requirement and is not at all in danger of falling into level 2. Airplane without yaw stiffness compensation, as in DIR law, is lack of the ability to turn itself into the direction of the sideslip rapidly. Therefore, the sideslip triggered by lateral stick step input is higher. The maximum sideslip falls on the boundary that divides level 1 and level 2 regions, indicating degrades of flying quality.

The airplane with lateral-directional reconfiguration law still satisfies level 2 requirements of MIL-STD-1797A. This is equivalent to satisfying adequate requirements in HQRM. However, decays in lateral stability, esp. Dutch roll decays, are still found in closed loop characteristics. It is therefore attentions should be paid to handling qualities with lateral-directional reconfiguration laws when designing RSS commercial aircraft.

5. Conclusions

Normal load factor feedback signals is lost in longitudinal direct control law, therefore can not provide pitch stiffness compensation. This results in degraded short period frequency qualities.

Degrading of lateral-directional flight control law leads to changes in spiral mode and Dutch roll characteristics. Alternative law, with the reduction of roll axis PI structure, is prone to a divergent spiral motion, although it does not lead to a down grade of spiral mode quality. Lateral direct law is prone to reduced yaw stiffness and damping. Although the damping is still sufficient, reduced stiffness leads to larger amplitude of sideslip motion that is associated with lateral control input. The sideslip excursion leads to down grade of flying qualities.

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